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香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九八八年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

數學
Mathematics

評卷參考
Marking Scheme

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本評卷參考並非標準答案，故極不宜
落於學生手中，以免引起誤會。

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It is highly undesirable that this
marking scheme should fall into the
hands of students. They are likely
to regard it as a set of model
answers, which it certainly is not.

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pleas from their students to have
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Solutions	Marks	Remarks
1. $a^2 - a - 6 = (a + 2)(a - 3)$ $a^3 + 8 = (a + 2)(a^2 - 2a + 4)$ Their L.C.M. = $(a + 2)(a - 3)(a^2 - 2a + 4)$ (= $a^4 - 3a^2 + 8a - 24$)	2A+1A 1M+1A 5	any 1 part correct Both exp. must first be factorized. at most 3 per paper at most 1 per question at most 1 for the same type of p.
2. (a) $\frac{\sin(180^\circ - \theta)}{\sin(90^\circ + \theta)} = \frac{\sin\theta}{\cos\theta}$ must be shown..... = $\tan\theta$	1A 1A	EXC
(b) $\sin^2(\pi - \theta) + \sin^2(\frac{3\pi}{2} + \theta)$ = $\sin^2\theta + \cos^2\theta$... OA = 1	1A 1A 5	For $\sin(\frac{3\pi}{2} + \theta) = -\cos\theta$
3. $2x^2 \geq 5x$ $2x^2 - 5x \geq 0$ $x(2x - 5) \geq 0$	1A 1A	Withhold 1 mark if '=' omitted. If solved by equation, no marks awarded unless answer correct.
Case (i) $x \geq 0$ and $2x - 5 \geq 0$ i.e. $x \geq \frac{5}{2}$ Case (ii) $x \leq 0$ and $2x - 5 \leq 0$ i.e. $x \leq 0$ Combining the two parts, we have $x \leq 0$ or $x \geq \frac{5}{2}$.	3A 5	Optional any 1 part without = , withhold 1 mark. For $x \leq 0$, $x \geq \frac{5}{2}$, 2 $x \leq 0$ and $x \geq \frac{5}{2}$ 1
4. (a) If $9x^2 - (k + 1)x + 1 = 0$ has equal roots, $(k + 1)^2 - 36 = 0$	1A	Alt. Solution: $(k+1)^2 - 36 = 0$ 1A
$k^2 + 2k - 35 = 0$	1A	$k + 1 = \pm 6$ 1A+1A
$(k - 5)(k + 7) = 0$ k = 5 or -7 both correct	1A	k = 5 or -7 1A k+1 = 6 1A only
(b) Putting k = -7 in (*) $9x^2 + 6x + 1 = 0$ $(3x + 1)^2 = 0$ $x = -\frac{1}{3}$	1M 1A 6	Sub. For negative value of k L.S. = $(3x + 1)^2$ $x = -\frac{1}{3}$ Sub. both for k=-7 and k=5 no marks

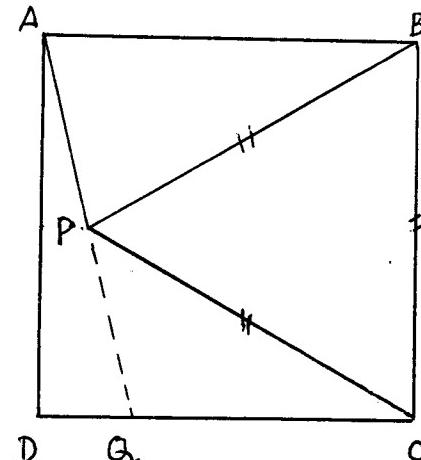
Solutions	Marks	Remarks
5. (a) Area of OABC = $\pi 10^2 \times \frac{100^\circ}{360^\circ}$ = 87.27 (corr. to 2 d.p.) (or 87.28)	1M 1A	
(b) Area of $\triangle OAC$ = $\frac{1}{2} \times 10 \times 10 \times \sin 100^\circ$ = 49.24 (corr. to 2 d.p.)	1M 1A	$\triangle = \frac{1}{2} AC \times OM$ $= \frac{1}{2} \times 15.3209 \times 6.4279$... 1M = 49.24 ... 1A
(c) Area of minor segment ABC = 87.27 - 49.24 = 38.03 (corr. to 2 d.p.) (or 38.04)	1M 1A 6	
6. $\log 2 = r$, $\log 3 = s$.		
(a) $\log 18 = \log 2 + \log 9$ = $\log 2 + \log 3^2$) = $\log 2 + 2\log 3$) = $r + 2s$	1A 1M 1A	For $18 = 2 \times 3^2$) $\log ab = \log a + \log b$ or) $\log a^2 = 2\log a$
(b) $\log 15 = \log 3 + \log 5$ = $\log 3 + \log \frac{10}{2}$ IA = $\log 3 + \log \frac{10}{2}$ = $\log 3 + \log 10 - \log 2$ = $1 - r + s$	1A 1A 1A 6	For $5 = \frac{10}{2}$ or $15 = \frac{30}{2}$
7. (a) The coordinates of the centre are given by only answer → correct 2A $x = -(-\frac{4}{2})$, $y = -\frac{10}{2}$ i.e. $x = 2$, $y = -5$	1M 1A	$(x-2)^2 + (y+5)^2 = \frac{25}{1} : k+4$
(b) As C touches the y-axis, without bracket its radius = 2 $4 + 25 - k = 2^2$ $k = 25$	1M+1A 1M 1A	OR Subs. $(0, -5)$ 1M $25 - 50 + k = 0$ 1M $k = 25$ 1A $r = \sqrt{4 + 25 - 25}$ 1M $= 2$ 1A
	6	OR Put $x = 0$, $y^2 + 10y + k = 0$ has equal roots. 1M $100 - 4k = 0$ $k = 25$ 1A $r = \text{etc.}$

Solutions

Marks

Remarks

8. (a) (i)



(ii) Since $\triangle PBC$ is equilateral, $\angle PBC = 60^\circ$
*angle written down
on the diagram*
 \rightarrow no method marks $\angle ABP = 90^\circ - 60^\circ = 30^\circ \dots\dots\dots$

$$\text{As } BA = BP, \angle PAB = \frac{1}{2}(180^\circ - 30^\circ) \\ = 75^\circ$$

$$\text{Since } AB \parallel DC, \angle PQC = 180^\circ - 75^\circ \\ = 105^\circ$$

1 $\angle PBC$ in order
 1 For P
 1 For Q (between D, C)

Follow through even if
diagram not accurate

$$\begin{aligned} & \angle PAD = 15^\circ \\ & \angle PQC = 90^\circ + 15^\circ \\ & = 105^\circ \end{aligned}$$

1M }
1A }

OR

1M }
1A }

7

(b) (i) $\triangle TCB$ is similar to $\triangle ACT$ because

both correct $\angle C$ is common.

1 mark $\angle BTC = \angle BAT$ (angle in alternate segment)
no need
 $\angle T$ no mark

$\triangle TCB \sim \triangle ACT$ (*AA.A.S.*) *no mark*

1 mark

$$(ii) \frac{AC}{CT} = \frac{CT}{BC} \dots\dots\dots$$

$$AC = \frac{6^2}{5} = 7.2 \quad \text{correct substitution}$$

$$\therefore AB = 7.2 - 5$$

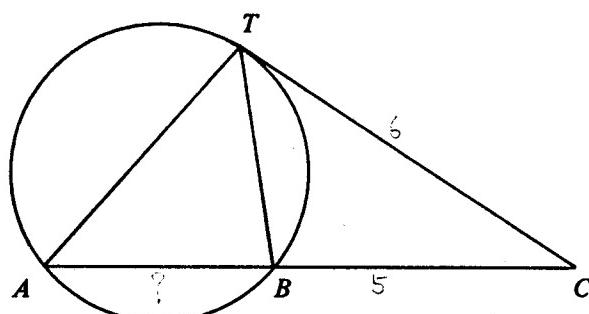
$$= 2.2 \quad (= \frac{11}{5}) \dots\dots\dots$$

1 \approx
 1 \cong
 Indication of 2 pairs
of equal angles. With-
held if proving con-
gruence.

Follow through even
if (b)(i) wrong.

1A

5



Solutions	Marks	Remarks
9. (a) Between 100 and 999, the smallest multiple of 7 is 105, the largest is 994.	1A <u>1A</u> <u>2</u>	
(b) The number of multiples is $\frac{994 - 105}{7} + 1$ must be correct. = 128	2M 1A	OR $994 = 105 + (n-1) \times 7$
The sum of these multiples = $105 + 112 + \dots + 994$ = $\frac{128}{2} [105 + 994]$ = 70336	2M <u>1A</u> <u>6</u>	
(c) The sum of all positive 3-digit integers = $100 + 101 + \dots + 999$ } or all correct = $\frac{900}{2} [100 + 999]$ = 494,550 The required sum = $494,550 - 70,336$ = 424,214	1 1A 1A 1M <u>1A</u> <u>4</u>	

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Solutions	Marks	Remarks
10. (a) Let $y = k_1x + k_2x^2$, where k_1 and k_2 are constants. Putting $x = 1$, $y = -5$; $x = 2$, $y = -8$, we have $k_1 + k_2 = -5$ $2k_1 + 4k_2 = -8$ Solving, $k_1 = -6$, $k_2 = 1$ $\therefore y = -6x + x^2$ Putting $x = 6$, we have $y = 0$.	2 1M 1A 1A 1A+1A 1A 8	For $y=kx+kx^2$ or $y = kx+x^2$ or $y = x+kx^2$ 1 $y=x+kx^2$ no marks marks ($y=k_1x+k_2x^2$)
(b) $y = -6x + x^2 = (x^2 - 6x + 9) - 9$ = $(x - 3)^2 - 9$ When $x = 3$, the value of y is least and the least value is -9.	1M 1A 1M+1A 4	Equality must hold. $y=(x+3)^2 - 9$ OA least value of y is -9 IM OA
11. (a) From the curve, (i) the median is 70 marks. (ii) the 1st quartile is 50 marks.) the 3rd quartile is 86 marks.) \therefore the interquartile range = $86 - 50$ = 36 marks	1A 1A 1M 1A 4	for either
(b) (i) From the curve, the number of prize-winners = 60. (ii) The probability that the student is a prize-winner = $\frac{60}{600}$ (= $\frac{1}{10}$). (iii) (1) The probability that both are prize-winners is $\frac{60}{600} \times \frac{59}{599}$ (= $\frac{59}{5990}$) (= 0.01) (2) The probability that both are not prize-winners = $\frac{540}{600} \times \frac{539}{599}$ (= $\frac{4851}{5990}$) (= 0.81) \therefore the probability that at least one is a prize-winner = $1 - \frac{4851}{5990}$ = $\frac{1139}{5990}$ (= 0.19)	1A 1M+1A 1M+1A 1A 1A 8	Accept $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ IM for product rule Accept $\frac{9}{10} \times \frac{9}{10}$ OR $\frac{9}{10} \times \frac{60}{599} + \frac{1}{10} \times \frac{540}{599}$ + $\frac{1}{10} \times \frac{59}{599}$ 1M+1A = $\frac{1139}{5990}$ 1A

Solutions

	Marks	Remarks
12. (a) L_3 is given by $\frac{x}{3} + \frac{y}{4} = 1$ i.e. $4x + 3y = 12$ (b) The three constraints are $y \leq 4$ $x \leq 3$ $4x + 3y \geq 12$	1M 1A 2	or 2-pt form, etc. Must be in this form.
(c) The line $x + 4y = c$ drawn in the diagram. From the diagram, P is greatest when $x = 3$, $y = 4$ and least when $x = 0$, $y = 0$. The greatest value of $P = 19$, the least value = 3.	1A 1A 1A 3 1M+1A	Withhold 1 mark if '=' omitted. or $4x + 3y - 12 \geq 0$. For 1A Drop of 2-3 verticle units for 10 horizontal units. OR Testing any vertices At $(3, 0)$, $P = 3$. At $(0, 4)$, $P = 16$. At $(3, 4)$, $P = 19$. 1A test 2 points only 1M
	1A 1A 1A 4	
(d) The line $2x - 3y + 3 = 0$ drawn in the diagram. The shaded region. P is least when $x = \frac{3}{2}$, $y = 2$. The least value = $\frac{19}{2}$ (= 9.5)	1A 1A 1A 3	±1 unit at $(1.5, 2)$, $(3, 3)$. Should be reasonably shaded. At $(3, 3)$, $P = 15$. At $(1.5, 2)$, $P = 9.5$.

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Solutions

Marks

Remarks

13. (a) $\frac{AB}{HB} = \tan\theta$	1M	any part in this question Wrong/no unit, pp-1. in the answer in each part 2 + 1
$HB = \frac{3}{\tan\theta} \text{ m}$	1A	
$\frac{DC}{KC} = \tan\theta$, $KC = \frac{2}{\tan\theta} \text{ m}$	1A	
	3	

(b) (i) $S_1 = \frac{6}{2} (3 + 2)$
 $= 15 \text{ m}^2$

(ii) $S_2 = \frac{6}{2} \left(\frac{3}{\tan\theta} + \frac{2}{\tan\theta} \right)$
 $= \frac{15}{\tan\theta} \text{ m}^2$

1A

1A

1A

Must show working..

$$\frac{15}{\frac{15}{\tan\theta}} = \tan\theta$$

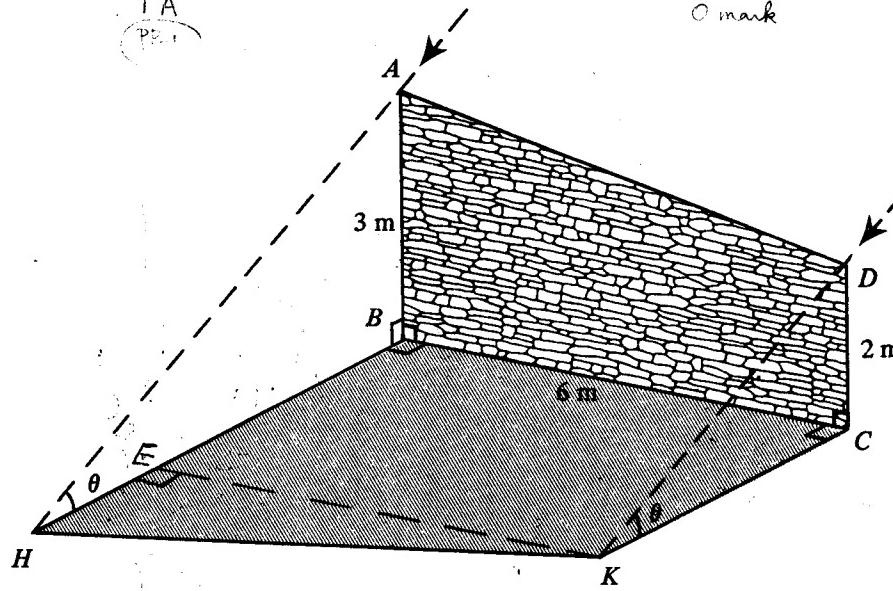
$$\tan\theta = \tan\theta$$

0 mark

$$\frac{15}{\frac{15}{\tan\theta}} = \frac{15}{\tan\theta}$$

$$= \tan\theta / A$$

(PP-1)

(c) Let $KE \perp BH$.

$$EK = BC = 6 \text{ (m)}$$

$$K? = 6 \quad \text{--- 2 marks}$$

1M

Construction of perpendicular line

$$\begin{aligned} HE &= \frac{3}{\tan\theta} - \frac{2}{\tan\theta} = \left(\frac{3}{\tan 30^\circ} - \frac{2}{\tan 30^\circ} \right) \text{ m } (= \sqrt{3}) \\ \therefore HK &= \sqrt{HE^2 + EK^2} \quad \begin{matrix} \text{IM} \\ \theta = 30^\circ \end{matrix} \\ &= \sqrt{(\sqrt{3})^2 + 6^2} \\ &= \sqrt{39} \text{ m} \end{aligned}$$

1M+1M
1M

$$\begin{aligned} HB &= 5.1961 \dots \text{ or } 5.2 \\ KC &= 3.464 \dots \text{ or } 3.5 \\ HE &= 1.732 \\ HK &= 6.24 \end{aligned}$$

1A

6

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Solutions	Marks	Remarks																																				
14. (a) (i) $x^3 - \frac{4}{3}x - 6 = 0$ can be written as $x^3 = \frac{4}{3}x + 6$. Consider the line $y = \frac{4}{3}x + 6$ It cuts the curve $y = x^3$ at $x = r$, where r lies between 2.0 and 2.1.	1M 1A+1A 1A	1A for equation 1A for line drawn, ±1 vertical division about (0, 6), (3, 10)																																				
(ii) Let $f(x) = x^3 - \frac{4}{3}x - 6$ $\left. \begin{array}{l} f(2) = -(-= -0.67) \\ f(2.1) = +(=0.46) \end{array} \right\}$ both correct but give 5 marks	1M	Correct change of sign.																																				
<table border="1"> <thead> <tr> <th>Interval</th> <th>Mid-value x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$2.000 < r < 2.100$</td> <td>2.050</td> <td>-(-=-0.12)</td> </tr> <tr> <td>$2.050 < r < 2.100$</td> <td>2.075</td> <td>+(=0.17)</td> </tr> <tr> <td>$2.050 < r < 2.075$</td> <td>2.063</td> <td>+(=0.02)</td> </tr> <tr> <td>$2.050 < r < 2.063$</td> <td>2.057</td> <td>-(-=-0.04)</td> </tr> <tr> <td>$2.057 < r < 2.063$</td> <td></td> <td></td> </tr> </tbody> </table> <p>$\therefore r = 2.06$ (correct to 2 d.p.)</p> <p><u>Alt. Solution:</u></p> $\left. \begin{array}{l} f(2) = - \\ f(2.5) = + \end{array} \right\} \dots \quad \text{Q.25 QM+OA}$ <table border="1"> <thead> <tr> <th>Interval</th> <th>Mid-value x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$2.000 < r < 2.500$</td> <td>2.250</td> <td>+</td> </tr> <tr> <td>$2.000 < r < 2.225$</td> <td>2.113</td> <td>+</td> </tr> <tr> <td>.</td> <td>.</td> <td>.</td> </tr> <tr> <td>.</td> <td>.</td> <td>.</td> </tr> <tr> <td>.</td> <td>.</td> <td>.</td> </tr> </tbody> </table> <p>$\therefore r = 2.06$ (correct to 2 d.p.)</p>	Interval	Mid-value x	f(x)	$2.000 < r < 2.100$	2.050	-(-=-0.12)	$2.050 < r < 2.100$	2.075	+(=0.17)	$2.050 < r < 2.075$	2.063	+(=0.02)	$2.050 < r < 2.063$	2.057	-(-=-0.04)	$2.057 < r < 2.063$			Interval	Mid-value x	f(x)	$2.000 < r < 2.500$	2.250	+	$2.000 < r < 2.225$	2.113	+	<p>1A 9</p> <p>1M+1A 1M</p> <p>1A</p> <p>1A</p>	<p>1M for choosing mid-value, 1A for correct sign.</p> <p>Next correct interval step.</p>
Interval	Mid-value x	f(x)																																				
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(b) Put $x = t + 1$	1A																																					
The given equation can be written as $3x^3 - 4x - 18 = 0$ or $x^3 - \frac{4}{3}x - 6 = 0$ By (a), the solution is $t = 2.06 - 1 \dots$ $= 1.06$ (correct to 2 d.p.)	1M 1A 3																																					

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14.

